Question 4

(b).

From the result of (a). we know that for optimization problem

The optimal solution is the-th quantile of the random variable as any point that satisfies the equation , where is the CDF of random variable .

According to the definition of CDF,.

For question (b), we are looking for the optimal solution of optimization problem

As is a function that map from to , and , expression and are equivalent. While the-th quantile of the random variable is the optimal solution for , given condition , the conditional -quantile of random variable is the optimal solution for expression .

Therefore, -quantile assess how much will change for distribution as change by 1 unit at quantile point for a given set of other covariates.

(c).

The goal is to find that

For , if

if

Therefore,

Define

For , where

So

Define

So

Define

So

Therefore

Therefore, we prove that